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Superconducting Cosmic Strings and the Photofission of ^{4}He in the Early Universe

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Abstract. We consider the photofission of primordial ^{4}He around 10^6 s after the bang by high energy photons produced from the catastrophic quenches of superconducting cosmic string loops present. Requiring that the photofission of ^{4}He not overproduce ^{3}He results in stringent constraints on the string tension and initial currents in superconducting loops, constraints which may preclude the structure formation scenario of Ostriker et al[1].

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Witten[2] has shown that cosmic strings can be superconducting, and others have shown that if they are, there are interesting and important consequences for structure formation in the Universe[1] and for the origin of the UHE cosmic rays[3]. The effect of both ordinary[4,5] and superconducting[5] cosmic strings on primordial nucleosynthesis, and the constraints that pertain, have also been discussed. Here we consider the photofission of ^{4}He into ^{3}He which results from superconducting cosmic string loops (SCCS) which catastrophically "quench" shortly after primordial nucleosynthesis ($t \sim 10^6$ sec, $T \sim 10^{-3}$ MeV), and the stringent constraints that follow by requiring that ^{3}He not be overproduced (summarized in Fig. 1).

First consider the bosonic SCCS loops that arise in a $U(1) \times U(1')$ gauge theory (Witten's toy model[2]) with Higgs potential:

$$V(\phi,\sigma) = -m_\phi^2|\phi|^2 + \lambda_\phi|\phi|^4/3! - m_\sigma^2|\sigma|^2 + \lambda_\sigma|\sigma|^4/3! + h|\phi|^2|\sigma|^2 + 3m_\phi^4/2\lambda_\phi \quad (1)$$

where the complex scalar field ϕ carries no ordinary (i.e., $U(1)$) charge and $U(1')$ charge q, while the complex scalar field σ carries no $U(1')$ charge and ordinary electric charge e. [We refer the reader to Ref. 6 for a detailed discussion of such strings.] Throughout we use units where $\hbar = c = k_B = 1$, so that $\alpha_{EM} = e^2/4\pi$ and Newton's constant $G_N \equiv m_{pl}^{-2}$ ($m_{pl} = 1.22 \times 10^{19}$ GeV).

A current-carrying SCCS loop of characteristic size R_o which forms in the early Universe oscillates with period $\tau \sim R_o$, and radiates both gravitational and electromagnetic radiation, eventually damping its motion and dissipating both its mass and electromagnetic field energy. The power radiated in gravitational waves and electromagnetic waves is respectively

$$P_{GW} \equiv \gamma_{GW} G\mu^2$$
$$P_{EM} \equiv \gamma_{EM} I^2$$

where μ is the string tension, I the electromagnetic current, and $\gamma_{GW} \simeq 50$ (Ref. 7) and $\gamma_{EM} \simeq 50$ are numerical constants. We will not address the origin of the primeval magnetic fields which are required to initiate currents in the SCCS loops, although this is a very interesting and important question[8].

Following Ostriker et al[1] (hereafter OTW) we define the critical current to be $I_{crit} = r(e\sqrt{\mu})$, and define the ratio of electromagnetic radiation to gravitational radiation power to be:

$$f = \frac{P_{EM}}{P_{GW}} = j^2(\gamma_{EM}/\gamma_{GW})e^2 r^2 (G\mu)^{-1} \tag{2}$$

where $j = I/I_{crit}$. OTW implicitly take the numerical constant r to be of the order of unity. A more detailed study[6] finds that for bosonic SCCS's

$$r \simeq [(m_\sigma^4/\lambda_\sigma)/(m_\phi^4/\lambda_\phi)]^{1/2}(\alpha/\beta)^{3/4} \tag{3}$$

where $\alpha \simeq m_\sigma^2/m_\phi^2$ and $\beta \simeq 3h/\lambda_\phi$. The stability of the $U(1) \times U(1')$ theory against the spontaneous breakdown of $U(1)$ requires[6]: $\beta \geq \alpha$ and $m_\sigma^4/\lambda_\sigma \leq m_\phi^4/\lambda_\phi$; and the existence of SCCS solutions requires in addition that $\beta \lesssim 0.5\alpha^{1.93}$, from which it follows that $r \lesssim 1$.

According to the numerical studies of Albrecht and Turok[9], a loop of characteristic size R_o forms by the self intersection(s) of infinite strands of string when the Universe has an age $t_o \simeq \epsilon^{-1} R_o$, where $\epsilon \simeq 0.2$. The mass energy of the typical loop which forms is $\simeq \beta\mu R_o$, where $\beta \simeq 9$. The number density of such loops (produced over a Hubble time) is: $n \simeq 2.6\nu\epsilon^{-3/2}t_o^{-3}$, where the numerical coefficient $\nu \simeq 0.01$.

As a loop oscillates, it radiates energy and shrinks in size; the current necessarily increases, $I = I_o R_o/R$, due to the conservation of the topological twist in the σ field which gives rise to the current. Therefore, it follows that $f = f_o(R_o/R)^2$. The critical current is achieved when the loop shrinks to a size j_o times its original size: $R_{crit} \simeq R_o j_o$. A loop radiating EM radiation and GW will reach this size at a time:

$$t_d \simeq (\beta/\gamma_{GW})(G\mu)^{-1}(1 - f_o^{1/2}\tan^{-1}(f_o^{-1/2}) - \delta)R_o \tag{4}$$

where $\delta = j_o - f_o^{1/2}\tan^{-1}(j_o f_o^{-1/2})$ is small and can be neglected so long as $I_o << I_{crit}$. When the current reaches I_{crit} (or at a slightly larger current), it becomes energetically favorable for the topological twist in the σ field to untwist, catastrophically dissipating the energy associated with the current into UHE particles[2,3,6,10] (typical energies of up to $\sim \mu^{1/2} \sim 10^{16}$ GeV). The EM field energy per length associated with the current is of the order of $\sim 7I^2$, which when $I = I_{crit}$ is a factor $f_I \simeq 7e^2r^2$ times the mass per length $(= \mu)$ of the string[6].

The precise nature of the particles which are released when a loop "quenches" and dissipates the EM field energy associated with the supercurrent is not known, and for our purposes is *probably* not relevant. We will be interested in the SCCS loops which quench when the age of the Universe is about 10^6 sec and the temperature is about 10^{-3} MeV. The Universe at this epoch is still radiation-dominated, with a baryon-to-photon ratio[11]: $\eta \simeq (4-7)\times 10^{-10}$. The only charged particles present are electrons, protons, and the light nuclei produced during primordial nucleosynthesis (^{4}He, ^{3}He, D, and ^{7}Li), and the electron to baryon ratio is about 7/8.

The thermalization of high energy particles during this epoch has been studied extensively by Lindley[12]. The general process involves each UHE particle producing an EM cascade, with the number of photons increasing as their energies decrease. The dominant processes for degrading the energy of superthermal photons are: $e^\pm$ pair production off thermal photons, and Compton scattering off the electrons present. For $E_\gamma T \gtrsim 0.02$ MeV^2 there are so many thermal photons above threshold for $e^\pm$ pair production that $e^\pm$ pair production is by far the dominant thermalization process. For photon energies of 20-40 MeV the cross sections for $\gamma + {}^4\mathrm{He} \to {}^3\mathrm{H} + \mathrm{p}$ and ^{3}He+n are each about $\sigma_{photo} \simeq 1.5$ mb, dropping rapidly for higher and lower energies (the threshold energy for these reactions is ~ 20 MeV). In order that the high energy photons produced by the cascade not rapidly degrade below 20 MeV before they can dissociate a ^{4}He nucleus, the temperature of the Universe should satisfy $T \simeq 0.02\mathrm{MeV}^2/(20-40\mathrm{MeV}) \simeq 10^{-3} - 5\times 10^{-4}$ MeV, thereby defining the epoch of significant photofission. During this epoch photofission only competes with Compton scattering off electrons. [At these energies $e^\pm$ pair production off protons and ^{4}He nuclei is less important than Compton scattering by about a factor of 10.] For photons of energy 20-40 MeV the Compton cross section is about $\sigma_c \simeq 25$ mb.

For a ^{4}He mass fraction of about 0.25 (what is expected from primordial nucleosynthesis and is observed to be the primordial abundance[11]), free electrons outnumber ^{4}He nuclei by about a factor of 10. Thus the probability that a 20-40 MeV photon produced by the cascade photodissociates a ^{4}He nucleus before undergoing a Compton scattering which degrades its energy is:

$$p_{photo} \simeq 2n(^4He)\sigma_{photo}/[2n(^4He)\sigma_{photo} + n(e^-)\sigma_c] \simeq 1.2\times 10^{-2}$$

[The factor of 2 arises because any ^{3}H produced eventually decays into ^{3}He.]

The inferred primordial abundance of ^{3}He is constrained by observations to be:

$n(^3\mathrm{He})/n(\mathrm{H}) \lesssim 10^{-4}$ (Ref. 11). Thus, the number of 20-40 MeV photons produced from SCCS quenches during the photofission epoch ($T \simeq 5 \times 10^{-4} - 10^{-3}MeV, t\simeq 1.3 \times 10^6$sec-$5.2 \times 10^6$sec) must be less than about $\sim (10^{-4}/p_{photo})$ per baryon. Taking η to be 5×10^{-10} and $p_{photo} \simeq 1.2 \times 10^{-2}$ implies the constraint:

$$n_{30}/n_\gamma \leq 4 \times 10^{-12}$$

where n_{30} is the number density of 20-40 MeV quench-produced photons during the photofission epoch.

We will now compute the expected value of n_{30}/n_γ in terms of $G\mu$ and f_o. The SCCS loops whose quenches can lead to photodissociation of ^{4}He were produced at

$$t_o \simeq (G\mu)(\gamma_{GW}/\beta\epsilon)(1 - f_o^{1/2}\tan^{-1} f_o^{-1/2})^{-1} t_{photo} \tag{5}$$

where $t_{photo} \simeq (1.3 - 5.2) \times 10^6$ sec, which for $f_o \lesssim 1$ and $G\mu \gtrsim 10^{-8}$ is greater than a few seconds after the bang. From a few seconds after the bang, when the $e^\pm$ pairs transferred their share of the entropy to the photons, until today, the number of photons should be approximately conserved. Taking the number density of loops at birth to be: $n_{loop} \simeq 2.6\nu\epsilon^{-3/2}t_o^{-3}$ and the number density of thermal photons to be $n_\gamma = 2\varsigma(3)T_o^3/\pi^2$, it follows that the ratio of these loops to thermal photons is:

$$n_{loop}/n_\gamma \simeq 5.0 \times 10^{-73}\nu\beta^{3/2}(\gamma_{GW}G\mu)^{-3/2}(1 - f_o^{1/2}\tan^{-1} f_o^{-1/2})^{3/2} \tag{6}$$

This ratio remains constant except for the decay of loops.

The quench of each loop ultimately produces $\sim f_I(R_{crit}/R_o)(\beta\mu R_o)/30$MeV 20-40 MeV photons available for the photodissociation of ^{4}He. Bringing all the factors together, the number of $\sim$ 30 MeV photons produced in the quench of a typical loop is:

$$N_{30} \simeq 4.1 \times 10^{70} r\gamma_{GW}^{3/2}\gamma_{EM}^{-1/2}(G\mu)^{5/2} f_o^{1/2}(1 - f_o^{1/2}\tan^{-1} f_o^{-1/2})^{-1} \tag{7}$$

Combining this factor with the number of loops per photon present, we find that the expected number of $\sim$ 30 MeV photons per thermal photon is

$$n_{30}/n_\gamma \simeq 0.02 r\nu\beta^{3/2}\gamma_{EM}^{-1/2} G\mu f_o^{1/2}(1 - f_o^{1/2}\tan^{-1} f_o^{-1/2})^{1/2} \tag{8}$$

Based upon the production of ^{3}He through the photodissociation of ^{4}He, we previously concluded that n_{30}/n_γ must be less than $\sim 4 \times 10^{-12}$, which leads to the following constraint to $G\mu$ and f_o:

$$rG\mu f_o^{1/2}(1 - f_o^{1/2}\tan^{-1} f_o^{-1/2})^{1/2} \lesssim 5.2 \times 10^{-9} A \qquad (9)$$

Here the factor $A = (0.01/\nu)(9/\beta)^{3/2}(\gamma_{EM}/50)^{1/2}[(n(^3He)/n(H))/10^{-4}](\eta/5\times10^{-10})$, and takes into account the scaling of our constraint with all the uncertain numerical quantities we have used. In the limits $f_o \lesssim 1$ and $f_o \gtrsim 1$, this constraint becomes:

$$\begin{aligned} r f_o^{1/2} G\mu &\lesssim 5.2 \times 10^{-9} A \qquad && (f_o \lesssim 1) \\ rG\mu &\lesssim 9.1 \times 10^{-9} A \qquad && (f_o \gtrsim 1) \end{aligned}$$

[In the simplified derivation outlined here we have only considered the SCCS loops produced over a Hubble time which quench around $\sim 10^6$ sec after the bang. We have also numerically computed the effects of all loops which quench around the time of the photofission epoch, taking into account the energy dependence of the photofission cross section and the competing processes which degrade energetic photons (i.e., $e^\pm$ pair production and Compton scattering), and arrive at essentially the same numerical result.]

In the case of fermionic SCCS loops, the quantity $r \simeq g/\pi^{3/2}$, from which it follows that $f_I \simeq g^2/40$, where g is the Yukawa coupling of the fermion species which gives rise to the superconductivity. By making these replacements in our analysis above we can obtain the analogous constraint on fermionic SCCS loops. The constraint that follows is essentially identical to constraint(9), with the quantity r replaced by $g/\pi^{3/2}$. For perturbativity to apply in the Higgs sector the Yukawa coupling g must necessarily be $\lesssim (4\pi)^{1/2}$; thus, in the fermionic case as well r must be less than order unity.

The explosive structure formation model of OTW[1] requires that $G\mu$ be of the order of 10^{-6}, and that f_o be in the range[13]: $10^{-2} \lesssim f_o \lesssim 10^2$. If indeed r is of order unity as OTW implicitly assume, our constraint based upon the photodissociation of ^{4}He into ^{3}He by quench produced high energy photons poses serious (if not fatal) difficulties for their model. We must, however, point out that r (or g), which is necessarily less than 1, could, by the proper selection of parameters in the Higgs potential, be significantly smaller than 1. Moreover the parameters ϵ, β, and ν (as well as γ_{GW} and γ_{EM}) which describe cosmic string loop production have only been determined numerically[9], and to date by only one group[14].

However, we note that r (or g) cannot be made arbitrarily small without other difficulties arising: if r (or g) is too small, it is not possible to have f_o sufficiently large without the initial current exceeding the critical current. Specifically, the requirement that $j_o \lesssim 1$ implies that:

$$f_o \lesssim 0.1(G\mu)^{-1}r^2 \tag{10}$$

(or $f_o \lesssim 3 \times 10^{-3}(G\mu)^{-1}g^2$). This constraint to f_o, together with the photofission constraint, are displayed in Fig. 1. From Fig. 1 it is clear that it is only possible to choose $f_0 \sim O(1)$ consistent with both constraints if $G\mu \lesssim 4 \times 10^{-6}$.

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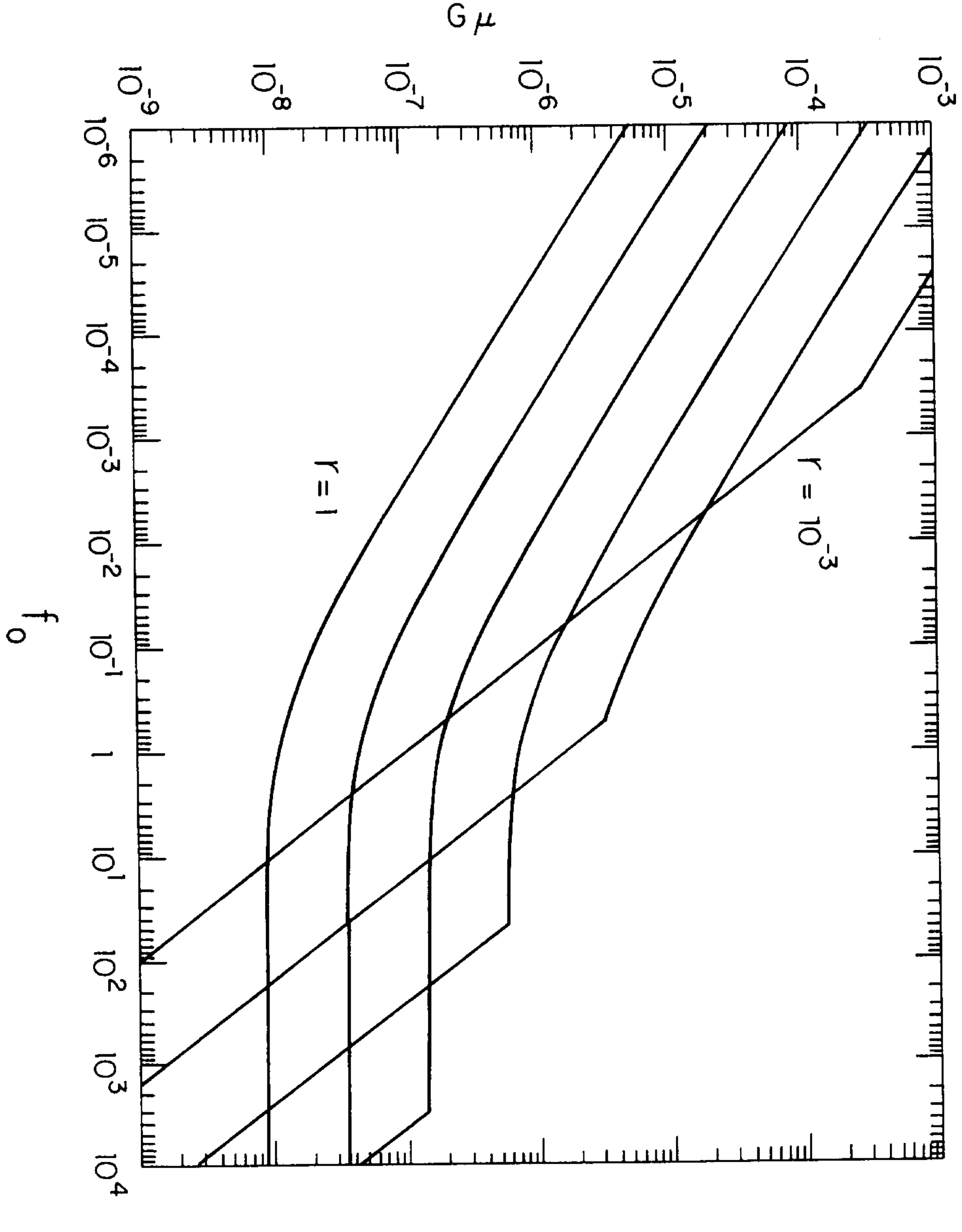
13. Our constraint to f_0 applies to loops which form around t_0 ($\gtrsim$ few sec after the bang). If the currents in the loops arise due to primeval magnetic fields which originate at very early times ($\ll 1$ sec), then f_0 should be approximately constant for all loops formed during the radiation-dominated epoch ($t \lesssim 10^{10}$ sec), and should decrease as 1/(cosmic scale factor) for loops formed during the subsequent matter-dominated epoch. The loops which give rise to explosive structure formation in the scenario of OTW form just after the Universe becomes matter-dominated. This means that our constraint for their loops is slightly *more* stringent.

14. Preliminary results of numerical work performed by Bouchet and Bennett seem to confirm the numerical results of Albrecht and Turok in a general way; however, there do seem to be some quantitative differences. See, F. Bouchet and D. Bennett, Fermilab preprint (1987).

Figure Caption

1. The maximum value of f_o (or $G\mu$) which is consistent with constraints (9, 10) for a given value of r as a function of $G\mu$ (or f_o). Curves are shown for $r = 1.0, 0.25, 6.3 \times 10^{-2}, 1.5 \times 10^{-2}, 4 \times 10^{-3}, 10^{-3}$. It is clear that irrespective of the value of r, f_o of the order of unity is only permitted if $G\mu \lesssim 4 \times 10^{-6}$.



$G\mu$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
f_o
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10^{1}
10^{2}
10^{3}
10^{4}
$r = 10^{-3}$
$r = 1$

- FIGURE 1 -